COMMUNICATIONS TO THE EDITOR

The Acceleration of the Surface of a Falling Film

Scott Lynn

Dow Chemical Company, Pittsburg, California

Scriven and Pigford (2) estimated the acceleration of the surface of a liquid film emerging from the feed slot of a wetted-wall column and the effect on the rate of gas absorption of the shifting velocity profile within the film. The authors considered the case of a relatively wide, horizontal slot feeding a vertical column. They assumed that the acceleration of the surface layer of the film was due only to gravity, neglecting the effects of surface tension and momentum transfer within the film. They then generalized their conclusions to cover all types of wetted-wall columns. It will be shown below that their results represent only an extreme limit, even for the special case which they treated. For the case of a column of the type described in reference 1, in which the slot width is of the same order as the steady state film thickness and in which the slot is also vertical, data obtained in a model study indicate that the acceleration of the surface of the film takes place in a distance about an order of magnitude smaller than would be estimated from the authors' equation.

The sketch in Figure 1 indicates the generalized problem on which the model study was based. The fluid is in full parabolic (laminar) flow at the

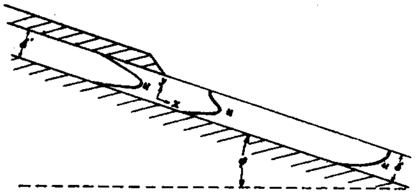

Fig. 1. Change in velocity profile of a laminar film emerging from between parallel plates into an open channel.

slot exit. In the region of interest the velocity distribution is changing to the half-parabolic distribution of laminar flow in an open channel. The surface layer is clearly being accelerated by the fluid under it. A force balance in the direction of flow on an element of volume yields

$$g\rho \sin \phi + \frac{\partial \tau}{\partial y} + \frac{\partial P}{\partial x} = \frac{D}{D\theta} \rho u \quad (1)$$

Outside the immediate vicinity of the slot exit $\partial P/\partial x$ is negligible. Furthermore from the definition of kinematic viscosity

$$\tau = \rho \nu \frac{\partial u}{\partial y} \quad (2)$$

so that at steady state Equation (1) becomes

$$g \sin \phi + \nu \frac{\partial^2 u}{\partial y^2} = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \quad (3)$$

Equation (3) can be put in dimensionless form by making the following substitutions:

$$\text{Let } X = \frac{x}{\delta} \quad U = \frac{u\delta}{\Gamma_v}$$

$$Y = \frac{y}{\delta} \quad V = \frac{v\delta}{\Gamma_v}$$

$$Re = \frac{4\Gamma_v}{\nu} \text{ (Reynolds number for open channel)}$$

The value of δ is found by solving Equation (3) with the right-hand side equal to zero:

$$\delta^3 = \frac{3\Gamma_v \nu}{g \sin \phi} \quad (4)$$

Substitution of this value in Equation (3) gives

$$\frac{4}{Re}\left(3 + \frac{\partial^2 U}{\partial Y^2}\right) = U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \quad (5)$$

The equation of continuity for two-dimensional, incompressible flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6)$$

so that

$$V = -\int_0^Y \frac{\partial U}{\partial X} dY \quad (7)$$

Substituting (7) and (5) and rearranging one gets

$$4\left(3 + \frac{\partial^2 U}{\partial Y^2}\right) = U \frac{\partial U}{\partial \left(\frac{X}{Re}\right)} - \frac{\partial U}{\partial Y} \int_0^Y \frac{\partial U}{\partial \left(\frac{X}{Re}\right)} dY \quad (8)$$

Equation (8) predicts that the velocity profile at any point downstream from the slot exit will depend only on X, Re, and the velocity profile at the slot exit. For the case described above the third parameter can be characterized by the ratio of the slot spacing to the equilibrium film thickness, δ'/δ.

In the model study, the slot spacing was 2.57 mm., the width of the flow channel was 99.4 mm., and the apparatus was inclined 6.00 deg. to the horizontal. The fluid flowed 100 mm. between the parallel plates before emerging into the open channel. The fluid used was a mixture of about 50% glycerine in water, the viscosity of which had been determined as a function of temperature. The fluid flow was metered with a gear pump. Lycopodeum powder was sprinkled on the surface of the liquid where it emerged from the slot. The spots of powder were then photographed by the light of a stroboscope which flashed 79.5 times/sec. ± 1%, and gave an average of six exposures per picture. The six successive values of x for a given spot of powder were plotted against a time scale with an arbitrary zero point. The slopes taken from the curves so obtained are plotted as the ratio of u_s to $3\Gamma_v/2$ vs. X/Re in Figure 2. The scatter of the data indicates the precision of the work. The closeness of approach of the points at high values of X/Re to unity indicates the accuracy. The approximation of Scriven and Pigford (2) is also shown.

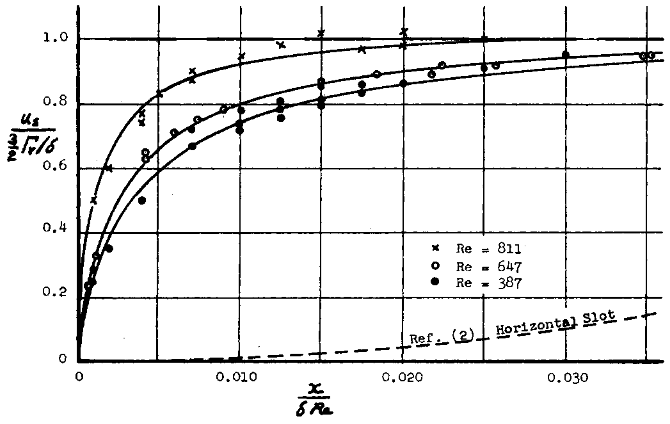

Fig. 2. Change in velocity of the surface of a laminar film emerging from between parallel plates into an open channel.

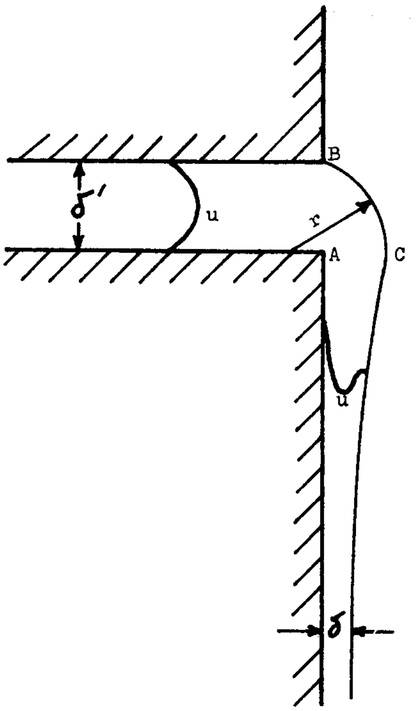

Fig. 3. Velocity changes in a falling film fed by a horizontal slot.

At Re = 647, the value of δ calculated from Equation (4) was just equal to the slot spacing. Thus the average velocity of the liquid within the slot is the same as that in the open channel. In the case of Re = 811, δ'/δ = 0.92; that is the film is thicker in the open channel, and hence the average velocity in the slot is higher than in the open channel. For Re = 387, δ'/δ = 1.19, and these conditions are reversed. These differences in average velocities would seem to be the reason for the differences in the rate of acceleration of the surace of the film.

From Figure 2 it is seen that the distance in which the film accelerates to 90% of its steady state velocity is about 13 δ for Re 647. A graphical integration of the solid curve drawn through these points can be made to estimate the actual contact time of the surface elements of the film. At *Re* 647 the additional contact time is equivalent to the time in which the film surface could travel an additional distance of 6 δ at the steady state velocity u_s. As is mentioned below, however, the authors have shown that the effect on gas absorption is even less than would be estimated on the basis of the additional contact time alone.

The flow situation treated by the authors is shown schematically in Figure 3. They assumed that since the liquid within the slot has no vertical component of velocity, the only force producing acceleration of the film down the wetted wall would be the force of gravity. This assumption is not even approximately correct; the assumption of zero velocity across the *A-C* plane would require an infinite film thickness at that point. Indeed a consideration of the forces acting within the film shows that the film thickness at *A-C* will generally be equal to or less than the slot spacing as long as δ' is greater than the steady state film thickness.

Equation (1) is valid for this flow situation if the appropriate coordinate system is used. However because of the curvature of the liquid surface the pressure of the liquid in the region *A-B-C* is still above atmospheric, and the pressure term in the equation cannot be neglected. Furthermore as in the case above the liquid layer on the surface is accelerated by the faster-flowing liquid within the film as soon as it emerges from the slot. Thus the surface of the film is caused to accelerate not only by the action of gravity but also by the effect of surface tension and by momentum transfer within the film.

A numerical example will serve to illustrate the order of magnitude of some of the quantities involved. The authors' calculations were based on a column of the type used by Vivian and Peaceman (3) in which a typical slot width was 1.3 mm. (Vivian and Peaceman actually used slots which sloped either up or down at about a 45-deg. angle, but this does not affect the basic argument presented here.) Consider a liquid flow rate of 25 cc./cm.-sec. of a fluid having a kinematic viscosity of 0.01 sq. cm./sec. and a surface tension of 75 dynes/cm. This corresponds to a Reynolds number of 1,000 on the column and would produce a film thickness of about 0.42 mm. and an average velocity of about 60 cm./sec. for the film at steady state. The pressure due to surface tension in a cylinder of fluid is equal to the surface tension divided by the radius of curvature. This pressure exerted over the area of the slot provides the force necessary to change the momentum of the stream from the horizontal to the vertical direction. The change in horizontal momentum in going from *A-B* to *A-C* is approximately $1.2\rho u^2_{av}$, that is 480 dynes/sq. cm. for this example. If the radius of curvature of the film were just equal to the slot width, the pressure in the region *A-B-C* would be about 580 dynes/sq. cm. above atmospheric. Since this pressure is more than enough to produce a vertical momentum at *A-C* equal to the horizontal momentum at *A-B*, it follows that the radius of curvature of the film will actually be somewhat greater than δ' and the film thickness at *A-C* will be somewhat less. (Since the film below *A-C* can only be getting thinner, the center of the radius of curvature must lie in the plane *A-C* or above it.)

The authors derived equations showing how the acceleration of the film at the inlet end of a wetted-wall column results in a lowered rate of absorption. Their equations consider not only the greater residence time of the surface of the film in this region but also the effect of the stretching of the film as it accelerates. That these two effects tend to be mutually compensating is apparent from the results obtained by the authors for a 4-cm. column at *Re* 1,000. They calculate a rate of absorption of about 84% of the rate which would obtain with instantaneous acceleration of the film. They calculate that the film will be accelerating over the entire length of the column, and the calculated contact time of an element of film surface is about twice what it would be with rapid acceleration. Based on contact time alone the calculated rate would have been only about 70% of the rate with rapid acceleration.

On the basis of the model study and the semiquantitative discussion presented above it appears that the authors' estimate of the distance over which the acceleration of the film takes place is five to ten times too great. It would thus seem likely that the effect of the acceleration would be noticeable only on columns shorter than 1 cm. This conclusion is in agreement with the work of Vivian and Peaceman (3), who found no effect of slot width or slot orientation in a column 4.2 cm. long. Of more importance under most circumstances in this range of column heights is the outlet end effect mentioned in reference *1*. This effect is the formation of a stagnant layer on the lower part of the falling film, which acts as an effective barrier to gas absorption.

ACKNOWLEDGMENT

The experimental work presented here was done in 1954 at the Laboratorium voor Fysische Technologie, Technische Hogeschool, Delft, Netherlands, under the direction of Professor H. Kramers. Peter Croockewit assisted with the experiments.

NOTATION

- g = acceleration due to gravity, cm./sec.^{2}
- P = pressure, dynes/sq. cm.
- u = velocity in x direction, cm./sec.
- u_s = velocity of surface in x direction, cm./sec.
- v = velocity in y direction, cm./sec.
- x = distance in direction of flow, cm.
- y = distance normal to direction of flow, cm.

Greek Letters

- Γ_v = volumetric flow rate, cc./cm.-sec.
- δ = film, thickness, cm.
- δ' = slot width, cm.
- θ = time, sec.
- ν = kinematic viscosity, sq. cm./sec.
- ρ = fluid density, gm./cc.
- τ = shear, dynes/sq. cm.
- ϕ = angle of inclination of flow channel to horizontal

LITERATURE CITED

1. Lynn, Scott, J. R. Straatemeier, and H. Kramers, *Chem. Eng. Sci.*, **4**, 58 (1955).
2. Scriven, L. E., and R. L. Pigford, *A.I.Ch.E. Journal*, **4**, 382 (1958).
3. Vivian, J. E., and D. W. Peaceman, *ibid.*, **2**, 437 (1956).

Dear Editor:

It has come to my attention that Dr. Joseph Joffe has published a figure relating minimum compressibility factor to reduced temperature.

Our note on page 171 of the March, 1960, issue of the *A.I.Ch.E. Journal*, in which we utilized the relationships of minimum compressibility factor to reduced temperature and pressure, should have given credit to Dr. Joffe for his earlier work with this relationship: Joffe, Joseph, *Chem. Eng. Progr.*, **45**, 160 (1949).

Very truly yours,

L. C. Case and
H. C. Weber
Purdue University
Lafayette, Indiana